The AC Method

The ac method is used for factoring trinomials of the form: \(ax^2 + bx + c\). The name ac comes from the fact that we multiply the coefficient of the first term “\(a\)” with the constant “\(c\)”.

**Step 1: Multiply \(a \times c\).**
Multiply the coefficient of the first term “\(a\)” with the constant “\(c\)”.
If there is a negative sign or subtraction sign in front of the coefficient or constant, then include that sign as part of the respective variable “\(a\)” or “\(c\)”.

Examples:
- \(12x^2 + 17x + 6\) \(a = 12\), \(c = 6\), \(a \times c = 72\)
- \(15x^2 + x - 2\) \(a = 15\), \(c = -2\), \(a \times c = -30\)
- \(20x^2 - 23x + 6\) \(a = 20\), \(c = 6\), \(a \times c = 120\)
- \(7x^2 - 3x + 3\) \(a = 7\), \(c = 3\), \(a \times c = 21\)

**Step 2: Write out the prime factorization for \(a \times c\).**
Write out \(a \times c\) as the product of their prime factors.

Examples:
- \(12x^2 + 17x + 6\) \(a \times c = 2 \times 2 \times 3 \times 3\)
- \(15x^2 + x - 2\) \(a \times c = -1 \times 2 \times 3 \times 5\)
- \(20x^2 - 23x + 6\) \(a \times c = -1 \times -1 \times 2 \times 2 \times 3 \times 5\)
- \(7x^2 - 3x + 3\) \(a \times c = 3 \times 7\)

**Note:** If \(a \times c\) is a negative number then include \((-1)\) as a factor. If the last term is positive and the middle term is negative then include two factors of \((-1)\).

**Step 3: Group the prime factors.**
Group the prime factors into two groups so that when the product of group one is added to the product of group two the result is the coefficient of the middle term: “\(b\)”. 

**Note 1:** Not all trinomials are factorable. If you cannot group the prime factors so that the product of group one plus the product of group two equals the middle term: “\(b\)”, then the trinomial is prime.

**Note 2:** Keep in mind that if the coefficient of the middle term is odd, then one group must be even and the other odd.

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<tr>
<th>Odd</th>
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<tbody>
<tr>
<td>+ Even</td>
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<td>Odd</td>
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</tbody>
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Note 3: If the coefficient of the middle term is even, then both groups must be even or both groups must be odd.

\[
\begin{array}{cccc}
    \text{Even} & \text{Odd} & \text{Even} & \text{Odd} \\
    + \text{Even} & + \text{Odd} & - \text{Even} & - \text{Odd} \\
    \text{Even} & \text{Even} & \text{Even} & \text{Even}
\end{array}
\]

Example:

\[12x^2 + 17x + 6\]
\[a \cdot c = 72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3\]
Group 1: \(2 \cdot 2 \cdot 2 = 8\) 
Group 2: \(3 \cdot 3 = 9\)
Group 1 + Group 2 = 8 + 9 = 17 = \(b\): the coefficient of the middle term

\[15x^2 + x - 2\]
\[a \cdot c = -30 = -1 \cdot 2 \cdot 3 \cdot 5\]
Group: \(2 \cdot 3 = 6\) 
Group 2: \(-1 \cdot 5 = -5\)
Group 1 + Group 2 = 6 + (-5) = 1 = \(b\): the coefficient of the middle term

\[20x^2 - 23x + 6\]
Group 1: \(-2 \cdot 2 \cdot 2 = -8\) 
Group 2: \(-3 \cdot 5 = -15\)
Group 1 + Group 2 = -8 + -15 = -23 = \(b\): the coefficient of the middle term

Note: If the last term is positive, both groups will have the same sign as the middle term.

\[7x^2 - 3x + 3\]
\[a \cdot c = 21 = 3 \cdot 7\]
There is no way to add these factors to get \((-3)\): the middle term. Therefore the trinomial is prime.

Step 4: Determine the replacement factors.
Use the factors from step 3 to determine the replacement factors for the middle term.

Examples:

\[12x^2 + 17x + 6\]
Middle term= 17x = 8x + 9x

\[15x^2 + x - 2\]
Middle term= 1x = 6x - 5x

\[20x^2 - 23x + 6\]
Middle term= -23x = -8x - 15x
**Step 5: Replace the middle term.**
Replace the middle term with the replacement factors determined in step 4.

Examples:
12\(x^2 + 17x + 6\)  15\(x^2 + x - 2\)  20\(x^2 - 23x + 6\)
12\(x^2 + 8x + 9x + 6\)  15\(x^2 + 6x - 5x - 2\)  20\(x^2 - 8x - 15x + 6\)

**Step 6: Factor by grouping.**
Now that we have all four terms we can use factor by grouping. Refer to the separate handout on factor by grouping to see how this is done.

**Examples: Start to finish.**

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15x^2 + 38x + 24
\]

\[
a \cdot c = 15 \cdot 24 = 360 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5
\]

Group 1: \(2 \cdot 2 \cdot 5 = 20\)  
Group 2: \(2 \cdot 3 \cdot 3 = 18\)

Group 1 + Group 2 = 20 + 18 = 38 = \(b\): coefficient of the middle term

Replacement factor: middle term = 38\(x = 20x + 18x\)

Ungrouped: 15\(x^2 + 20x + 18x + 24\)

Grouped: \((15x^2 + 20x) + (18x + 24)\)

Factor out the GCF: 5\((3x + 4) + 6(3x + 4)\)

Factor out the common binomial: \((3x + 4)(5x + 6)\)

\[
17x^2 - 50x - 3
\]

\[
a \cdot c = 17 \cdot 3 = -51 = -1 \cdot 3 \cdot 17
\]

Group 1: \(-1 \cdot 17 \cdot 3 = -51\)  
Group 2: \(2: 1\)

Group 1 + Group 2 = \(-51 + 1+ = -50 = \(b\): coefficient of the middle term

Replacement factor: middle term = \(-50x = -51x + 1x\)

Ungrouped: 17\(x^2 - 51x + 1x - 3\)

Grouped: \((17x^2 - 51x) + (1x - 3)\)

Factor out the GCF: 17\(x(x - 3) + 1(x - 3)\)

Factor out the common binomial: \((x - 3)(17x + 1)\)