Graphing Trigonometric Functions

\[ y = a \sin bx \quad \text{or} \quad y = a \cos bx \]

amplitude = |a|

period = \(\frac{2\pi}{|b|}\)

\[ y = a \csc bx \quad \text{or} \quad y = a \sec bx \]

amplitude = Does Not Exist

period = \(\frac{2\pi}{|b|}\)

\[ y = a \tan bx \quad \text{or} \quad y = a \cot bx \]

amplitude = DNE

period = \(\frac{\pi}{|b|}\)

\[ y = a \sin(bx + c) \quad \text{or} \quad y = a \cos(bx + c) \]

amplitude = |a|

period = \(\frac{2\pi}{|b|}\)

phase shift = \(-\frac{c}{b}\) (shifts left if the result is negative & right if positive)

\[ y = a \csc(bx + c) \quad \text{or} \quad y = a \sec(bx + c) \]

amplitude = DNE

period = \(\frac{2\pi}{|b|}\)

phase shift = \(-\frac{c}{b}\) (shifts left if the result is negative & right if positive)

\[ y = a \tan(bx + c) \quad \text{or} \quad y = a \cot(bx + c) \]

amplitude = DNE

period = \(\frac{\pi}{|b|}\)

phase shift = \(-\frac{c}{b}\) (shifts left if the result is negative & right if positive)
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Graphing the Sine and Cosine Functions (no phase shift)
To graph \( y = a \sin bx \) or \( y = a \cos bx \), with \( b > 0 \):

1. Find the period, \( \frac{2\pi}{b} \). Start at 0 on the x-axis and lay off a distance of \( \frac{2\pi}{b} \).
2. Divide the interval into four equal parts.
3. Evaluate the function for each of the five x-values resulting from Step 2. The points will be maximum points, minimum points, and x-intercepts.
4. Plot the points found in Step 3, and join them with a sinusoidal curve with amplitude \( |a| \).
5. Draw the graph over additional periods, as needed.

Graphing the Tangent and Cotangent Functions
To graph \( y = a \tan bx \) or \( y = a \cot bx \), with \( b > 0 \):

1. The period is \( \frac{\pi}{b} \). To locate two adjacent vertical asymptotes, solve the following equations for x:
   - For \( y = a \tan bx \): \( bx = -\frac{\pi}{2} \) and \( bx = \frac{\pi}{2} \)
   - For \( y = a \cot bx \): \( bx = 0 \) and \( bx = \pi \)
2. Sketch the two vertical asymptotes found in Step 1.
3. Divide the interval formed by the vertical asymptotes into four equal parts.
4. Evaluate the function for the first-quarter point, midpoint, and third-quarter point, using the x-values found in Step 3.
5. Join the points with a smooth curve, approaching the vertical asymptotes. Draw additional asymptotes and periods of the graph as necessary.

Graphing General Sine and Cosine Functions (with phase shift)
To graph the general function \( y = c + a \sin (bx - d) \) or \( y = c + a \cos (bx - d) \), where \( b > 0 \), follow these steps.

1. Find an interval whose length is one period \( \left( \frac{2\pi}{b} \right) \) by solving the compound inequality
   \[ 0 \leq bx - d \leq 2\pi \]
2. Divide the interval into four equal parts.
3. Evaluate the function for each of the five x-values resulting from Step 2. The points will be maximum points, minimum points, and points that intersect the line \( y = c \) ("middle" points of the wave).
4. Plot the points found in Step 3, and join them with a sinusoidal curve.
5. Draw the graph over additional periods, to the right and to the left, as needed.

The amplitude of the function is \( |a| \). The vertical translation is \( c \) units up if \( c > 0 \), \( |c| \) units down if \( c < 0 \).

Graphing the Cosecant and Secant Functions
To graph \( y = a \csc bx \) or \( y = a \sec bx \), with \( b > 0 \), follow these steps.

1. Graph the corresponding reciprocal function as a guide, using a dashed curve.
2. Sketch the vertical asymptotes. They will have equations of the form \( x = k \), where \( k \) is an x-intercept of the graph of the guide function.
3. Sketch the graph of the desired function by drawing the typical U-shaped branches between the adjacent asymptotes. The branches will be above the graph of the guide function when the function values are positive, and below the graph of the guide function when the function values are negative.

Inverse Trig Functions:

\[ y = \sin^{-1} x \quad y = \cos^{-1} x \quad y = \tan^{-1} x \]