Del Mar College
Core Curriculum Application
(to add Course to Core)

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How does this course meet the definition of the selected Foundational Component Area (definitions available on pp. 3-4)? If applying only for Component Area Option, which Foundational Component Area definition(s) does the course fit, and how?

The ACGM description for Math 2413, Calculus I, is “Limits and continuity; the Fundamental Theorem of Calculus; definition of the derivative of a function and techniques of differentiation; applications of the derivative to maximizing or minimizing a function; the chain rule, mean value theorem, and rate of change problems; curve sketching; definite and indefinite integration of algebraic, trigonometric, and transcendental functions, with an application to calculation of areas.” MATH 2413 focuses on quantitative literacy and involves the understanding of key mathematical concepts and the application of appropriate quantitative tools to everyday experiences, such as maximization, minimization, physical science, biology, population growth, volume, area, and work. Additional topics for quantitative tools are available in textbook indices of applications.

**ACGM Student Learning Outcomes**

Upon successful completion of this course, students will:

1. Develop solutions for tangent and area problems using the concepts of limits, derivatives, and integrals.
2. Draw graphs of algebraic and transcendental functions considering limits, continuity, and differentiability at a point.
3. Determine whether a function is continuous and/or differentiable at a point using limits.
4. Use differentiation rules to differentiate algebraic and transcendental functions.
5. Identify appropriate calculus concepts and techniques to provide mathematical models of real-world situations and determine solutions to applied problems.
6. Evaluate definite integrals using the Fundamental Theorem of Calculus.
7. Articulate the relationship between derivatives and integrals using the Fundamental Theorem of Calculus.
Attach an outline of major topics and major student assignments to demonstrate how the proposed course will prepare students to achieve the required Core Objectives for the selected Foundational Component Area(s).

See attached course syllabus and pages showing sample problems which address Core Objectives. A brief summary of major student assignments is included.

List the activities/measures that will be used to address and assess the required Core Objectives (definitions of Core Objectives available on p. 5, required Core Objectives for each Foundational Component Area found on p. 6). If applying for Component Area Option only, identify activities/measures for Critical Thinking, Communication Skills, and one other Core Objective.

Overview: Mathematics faculty members have contacted Ms. Risé Knight for ideas on assessment measures, and she has provided helpful input. Mathematics will primarily use exams and graded assignments to assess student mastery of the required Core Objectives. Since almost all math faculty members use the MyMathLab software package, common graded assignments can be given to students to assess mastery of Critical Thinking, Communication Skills, and Empirical & Quantitative Skills. With the MyMathLab program, each student’s problems will be different and grading will be uniform among all sections. Uniform partial credit will be awarded by the program for those problems where partial credit is appropriate. Members of the Math Assessment Committee will make assignments or quizzes that assess student learning outcomes and support the Core Objectives. The committee will then “share” the assignments with other faculty so that a large sample is achieved. For the few faculty members who teach Math 2413 and do not use MyMathLab, the assignments and quizzes can be printed out so that those teachers can periodically administer the assignments or quizzes to their classes.

Communication Skills:
1. MyMathLab Graphing Assessment Quizzes/Assignments – assess the ability of students to develop, interpret and express mathematical ideas effectively through visual and written communication.
2. MyMathLab Short Answer and Fill-In-The-Blank Quizzes/Assignments – assess the ability of the student to communicate mathematical ideas through written communication.
3. Question and Answer sessions in class – reinforce the ability of students to develop and express mathematical ideas effectively through oral communication.
4. Classroom Presentations – reinforce the ability of students to express mathematical ideas effectively through written, oral and visual communications.
5. Common Student Learning Outcomes Exams administered near end of term – Various problems, such as those on finance, investment, cost-revenue-profit analysis, assess written communication.
6. Homework Assignments (Approximately 24 per semester) – reinforce oral, visual, and written communication skills
7. Classroom Discussions – reinforce oral communications

Empirical & Quantitative Skills:
1. MyMathLab Assessment Quizzes/Assignments – assess the ability of students to use calculus techniques to collect, manipulate and analyze data and draw informed conclusions.
2. Common Student Learning Outcomes Exams administered near end of term – assess the ability of students to use calculus techniques to collect, manipulate and analyze data and draw informed conclusions.
3. Approximately 24 Homework Assignments – reinforce the ability of students to use techniques of calculus to collect, manipulate and analyze data and draw informed conclusions.

Critical Thinking:
1. Homework Assignments (Approximately 24 per semester) – reinforce all 6 aspects of Critical Thinking: creative thinking, innovation, inquiry, analysis, evaluation and synthesis.
2. MyMathLab Assessment Quizzes/Assignments – assess the ability of students to use techniques of calculus to analyze the components of a problem, evaluate the relevancy of those components, and synthesize the information to arrive at a problem solution. (Aspects 4, 5, 6)
3. Common Student Learning Outcomes Exam administered near end of term – assess the ability of students to use techniques of calculus to analyze the components of a problem, evaluate the relevancy of those components, and synthesize the information to arrive at a problem solution. (Aspects 4, 5, 6)

At which level (introduced, reinforced, mastered and assessed) will each aspect of the required Core Objectives be addressed? What is the achievement target for each that will be assessed?

Communication Skills:
Visual Communication Skills will be regularly assessed. Oral Communication Skills will be reinforced. Written Communication Skills will be primarily reinforced, but may be occasionally assessed.

1. MyMathLab Graphing Assessment Quizzes/Assignments – assess the ability of students to develop, interpret and express mathematical ideas effectively through visual communication. These computer-generated, individualized quizzes and assignments are graded by the software package to ensure uniformity. The target achievement level is for the average of the assignment or quiz to be a 70 or better.
2. MyMathLab Short Answer and Fill-In-The-Blank Quizzes/Assignments – assess the ability of the student to communicate mathematical ideas through written communication. These computer-generated, individualized quizzes and assignments are graded by the software package to ensure uniformity. The target achievement level is for the average of the assignment or quiz to be a 70 or better.
3. Question and Answer sessions in class – reinforce the ability of students to develop and express mathematical ideas effectively through oral communications.
4. Classroom Presentations in class – reinforce the ability of students to express mathematical ideas effectively through written, oral and visual communications.
5. Common Student Learning Outcome Exams administered near end of term – Short answer problems can be included on the SLO exams, such as those on finance, investments, amortization, maximization and probability to assess written communication. The achievement target will be for at least 70% of those questions designed to assess mastery of a student learning outcome are answered correctly by the students taking the exam. For those problems requiring written responses, the mathematical calculations must be correct in order for the students to receive any credit. If the mathematics is correct, then the sentence structure and grammar are evaluated by the instructor. If the mathematical calculations are correct and the answer is well-written, the student receives 2 points for the problem; if the calculations are correct but poorly written, then the student receives only 1 point. Graphing problems on the SLO Exams will assess visual communication skills.
6. Homework Assignments (Approximately 24 per semester) – reinforce oral, visual, and written communication skills.
7. Classroom Discussions – reinforce oral communications

Empirical & Quantitative Skills:
All three aspects, collect data, manipulate data, and analyze data to draw informed conclusions, will be assessed.

1. MyMathLab Assessment Quizzes/Assignments – assess the ability of students to use calculus techniques to collect, manipulate and analyze data and draw informed conclusions. These computer-generated, individualized quizzes and assignments are graded by the software package to ensure uniformity. The target achievement level is for the average of the assignment or quiz to be a 70 or better.

2. Common Student Learning Outcomes Exams administered near end of term – assess the ability of students to use calculus techniques to collect, manipulate and analyze data and draw informed conclusions. The achievement target will be for at least 70% of those questions designed to assess mastery of a student learning outcome are answered correctly by the students taking the exam.

3. Approximately 24 Homework Assignments – reinforce the ability of students to use the techniques of calculus to collect, manipulate and analyze data and draw informed conclusions.

Critical Thinking:
The first three aspects of Critical Thinking (Creative Thinking, Innovation and Inquiry) will be reinforced. The other three aspects (Analysis, Evaluation and Synthesis) will be assessed.

1. Homework Assignments (Approximately 24 per semester) – reinforce all 6 aspects of Critical Thinking: creative thinking, innovation, inquiry, analysis, evaluation and synthesis.

2. MyMathLab Assessment Quizzes/Assignments – assess the ability of students to use calculus techniques to analyze the components of a problem, evaluate the relevancy of those components, and synthesize the information to arrive at a problem solution. (Aspects 4, 5, 6) These computer-generated, individualized quizzes and assignments are graded by the software package to ensure uniformity. The target achievement level is for the average of the assignment or quiz to be a 70 or better.

3. Common Student Learning Outcome Exams administered near end of term – assess the ability of students to use calculus techniques to analyze the components of a problem, evaluate the relevancy of those components, and synthesize the information to arrive at a problem solution. (Aspects 4, 5, 6) The achievement target will be for at least 70% of those questions designed to assess mastery of a student learning outcome are answered correctly by the students taking the exam.

Identify any prerequisite REM (reading, English, and math) levels required for this course.
Reading 3
English 1
Math 3
List prerequisite and/or co-requisite courses (if any)
MATH 1314 and MATH 1316 or permission of mathematics department chair

Contact
For questions or comments concerning this application, please contact Anthony or Timothy Precella at extension 1906.

Signature Indicates Approval:

Department Chair: [Signature]

[Signature]

Division Dean: [Signature]

Core Curriculum Committee Recommends Approval:

Yes [ ] No [ ]

Provost Approval: [Signature]

Date: 10/5/17

Date: 11/15/12
Attach an outline of major topics and major student assignments to demonstrate how the proposed course will prepare students to achieve the required Core Objectives for the selected Foundational Component Area(s).

Below are the major topics of Calculus I. See the attached Syllabus for an exact listing of topics and the corresponding Core Objectives.

**Differential Calculus**
- Limits and Their Applications
- The Formal Definition of Limit
- One-sided Limits & Limits Involving Infinity
- Rates of Change (Average & Instantaneous)
- Tangent Lines & Derivatives
- Techniques for Finding Derivatives
- Derivatives of Trigonometric Functions
- Implicit Differentiation

**Applications of Derivatives**
- Derivatives and Graphs
- The Second Derivative
- Optimization Applications (Max/Min)
- Curve Sketching

**Integral Calculus**
- Summation
- Antiderivatives
- Indefinite Integrals
- Substitution
- Area and the Definite Integral
- The Fundamental Theorem of Calculus
- Applications of Integrals: Volume, Arc Length and Surface Area

**Summary of Major Student Assignments**

At least two major exams and a comprehensive final exam are administered in each section of Math 2413.

The Student Learning Outcomes and the Core Objectives of Critical Thinking, Empirical and Quantitative Analysis and Visual Communication Skills will be assessed by means of embedded test questions and by MyMathLab assignments and projects covering the student learning outcomes and the core objectives.

A MyMathLab-generated graphing assignment/quiz will be used to assess students' visual communication skills.

Daily homework assignments covering a minimum of 20 topics (see attached syllabus) will be made.
Sample Homework Problems That Address Communication Skills

51. Suppose $\lim_{x \to 0} f(x) = 1$ and $\lim_{x \to 0} g(x) = -5$. Name the rules in Theorem 1 that are used to accomplish steps (a), (b), and (c) of the following calculation.

\[
\lim_{x \to 0} \frac{2f(x) - g(x)}{(f(x) + 7)^{2/3}} = \lim_{x \to 0} \frac{2f(x) - g(x)}{(f(x) + 7)^{2/3}}
\]

(a) \[
= \frac{\lim_{x \to 0} 2f(x) - \lim_{x \to 0} g(x)}{(\lim_{x \to 0} (f(x) + 7))^{2/3}}
\]

(b) \[
= \frac{2 \lim_{x \to 0} f(x) - \lim_{x \to 0} g(x)}{(\lim_{x \to 0} f(x) + \lim_{x \to 0} 7)^{2/3}}
\]

(c) \[
= \frac{(2)(1) - (-5)}{(1 + 7)^{2/3}} = \frac{7}{4}
\]

69. Let $G(x) = \frac{x + 6}{x^2 + 4x - 12}$.

a. Make a table of the values of $G$ at $x = -5.9, -5.99, -5.999$, and so on. Then estimate $\lim_{x \to -6} G(x)$. What estimate do you arrive at if you evaluate $G$ at $x = -6.1, -6.01, -6.001, \ldots$ instead?

b. Support your conclusions in part (a) by graphing $G$ and using Zoom and Trace to estimate $y$-values on the graph as $x \to -6$.

c. Find $\lim_{x \to -6} G(x)$ algebraically.

Theory and Examples

75. If $x^4 \leq f(x) \leq x^2$ for $x$ in $[-1, 1]$ and $x^2 \leq f(x) \leq x^4$ for $x < -1$ and $x > 1$, at what points $c$ do you automatically know $\lim_{x \to c} f(x)$? What can you say about the value of the limit at these points?
76. Suppose that \( g(x) \leq f(x) \leq h(x) \) for all \( x \neq 2 \) and suppose that
\[
\lim_{x \to 2} g(x) = \lim_{x \to 2} h(x) = -5.
\]
Can we conclude anything about the values of \( f, g, \) and \( h \) at \( x = 2 \)? Could \( f(2) = 0 \)? Could \( \lim_{x \to 2} f(x) = 0 \)? Give reasons for your answers.

**Finding Deltas Graphically**
In Exercises 7–14, use the graphs to find a \( \delta > 0 \) such that for all \( x \)
\[
0 < |x - x_0| < \delta \quad \Rightarrow \quad |f(x) - L| < \epsilon.
\]

**7.**
\[
y = 2x - 4
\]
\[
\begin{align*}
\text{\( f(x) \)} & = 2x - 4 \\
\text{\( x_0 \)} & = 5 \\
\text{\( L \)} & = 6 \\
\text{\( \epsilon \)} & = 0.2
\end{align*}
\]
\[\text{NOT TO SCALE}\]

**8.**
\[
y = -\frac{3}{2}x + 3
\]
\[
\begin{align*}
\text{\( f(x) \)} & = -\frac{3}{2}x + 3 \\
\text{\( x_0 \)} & = -3 \\
\text{\( L \)} & = 7.5 \\
\text{\( \epsilon \)} & = 0.15
\end{align*}
\]
\[\text{NOT TO SCALE}\]
13. \[ f(x) = \frac{2}{\sqrt{-x}} \]
- \( x_0 = -1 \)
- \( L = 2 \)
- \( \epsilon = 0.5 \)

14. \[ f(x) = \frac{1}{x} \]
- \( x_0 = \frac{1}{2} \)
- \( L = 2 \)
- \( \epsilon = 0.01 \)

49. \( \lim_{x \to 0} x \sin \frac{1}{x} = 0 \)
10. Catching rainwater A 1125 ft$^3$ open-top rectangular tank with a square base $x$ ft on a side and $y$ ft deep is to be built with its top flush with the ground to catch runoff water. The costs associated with the tank involve not only the material from which the tank is made but also an excavation charge proportional to the product $xy$.

a. If the total cost is

$$c = 5(x^2 + 4xy) + 10xy,$$

what values of $x$ and $y$ will minimize it?

b. Give a possible scenario for the cost function in part (a).

12. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

13. Two sides of a triangle have lengths $a$ and $b$, and the angle between them is $\theta$. What value of $\theta$ will maximize the triangle’s area? (Hint: $A = \frac{1}{2}ab \sin \theta$.)
17. Designing a suitcase  A 24-in.-by-36-in. sheet of cardboard is folded in half to form a 24-in.-by-18-in. rectangle as shown in the accompanying figure. Then four congruent squares of side length \( x \) are cut from the corners of the folded rectangle. The sheet is unfolded, and the six tabs are folded up to form a box with sides and a lid.

a. Write a formula \( V(x) \) for the volume of the box.

b. Find the domain of \( V \) for the problem situation and graph \( V \) over this domain.

c. Use a graphical method to find the maximum volume and the value of \( x \) that gives it.

d. Confirm your result in part (c) analytically.

e. Find a value of \( x \) that yields a volume of 1120 in\(^3\).

f. Write a paragraph describing the issues that arise in part (b).

20. a. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 in. What dimensions will give a box with a square end the largest possible volume?

b. Graph the volume of a 108-in. box (length plus girth equals 108 in.) as a function of its length and compare what you see with your answer in part (a).
21. (Continuation of Exercise 20.)

a. Suppose that instead of having a box with square ends you have a box with square sides so that its dimensions are \( h \) by \( h \) by \( w \) and the girth is \( 2h + 2w \). What dimensions will give the box its largest volume now?

b. Graph the volume as a function of \( h \) and compare what you see with your answer in part (a).

14. **Cavalieri's principle**  A solid lies between planes perpendicular to the \( x \)-axis at \( x = 0 \) and \( x = 12 \). The cross-sections by planes perpendicular to the \( x \)-axis are circular disks whose diameters run from the line \( y = x/2 \) to the line \( y = x \) as shown in the accompanying figure. Explain why the solid has the same volume as a right circular cone with base radius 3 and height 12.
Sample Homework Problems That Address Critical Thinking

25. **Paper folding** A rectangular sheet of 8.5-in.-by-11-in. paper is placed on a flat surface. One of the corners is placed on the opposite longer edge, as shown in the figure, and held there as the paper is smoothed flat. The problem is to make the length of the crease as small as possible. Call the length $L$. Try it with paper.

a. Show that $L^2 = 2x^3/(2x - 8.5)$.

b. What value of $x$ minimizes $L^2$?

c. What is the minimum value of $L$?

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39. **Shortest beam** The 8-ft wall shown here stands 27 ft from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.

---

40. **Motion on a line** The positions of two particles on the $s$-axis are $s_1 = \sin t$ and $s_2 = \sin (t + \pi/3)$, with $s_1$ and $s_2$ in meters and $t$ in seconds.

a. At what time(s) in the interval $0 \leq t \leq 2\pi$ do the particles meet?

b. What is the farthest apart that the particles ever get?

c. When in the interval $0 \leq t \leq 2\pi$ is the distance between the particles changing the fastest?
In Exercises 41–44, find the volume of the solid generated by revolving each region about the y-axis.

41. The region enclosed by the triangle with vertices (1, 0), (2, 1), and (1, 1)

42. The region enclosed by the triangle with vertices (0, 1), (1, 0), and (1, 1)

43. The region in the first quadrant bounded above by the parabola $y = x^2$, below by the x-axis, and on the right by the line $x = 2$

41. A bead is formed from a sphere of radius 5 by drilling through a diameter of the sphere with a drill bit of radius 3.
   a. Find the volume of the bead.
   b. Find the volume of the removed portion of the sphere.

42. A Bundt cake, well known for having a ringed shape, is formed by revolving around the y-axis the region bounded by the graph of $y = \sin(x^2 - 1)$ and the x-axis over the interval $1 \leq x \leq \sqrt{1 + \pi}$. Find the volume of the cake.

43. Derive the formula for the volume of a right circular cone of height $h$ and radius $r$ using an appropriate solid of revolution.

44. Derive the equation for the volume of a sphere of radius $r$ using the shell method.
Revolution About the Axes

In Exercises 1–6, use the shell method to find the volumes of the solids generated by revolving the shaded region about the indicated axis.

1. \( y = 1 + \frac{x^2}{4} \)

2. \( y = 2 - \frac{x^2}{4} \)

3. \( y = \sqrt{2} \quad x = y^2 \)

4. \( y = \sqrt{3} \quad x = 3 - y^2 \)
# MATH 2413 CALCULUS I SYLLABUS with CORE OBJECTIVES

*Thomas' Calculus* by George Thomas, Maurice Weir, Joel Hass and Frank Giordano (12th Edition)

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Note: Oral communication is reinforced in all lecture classes through in-class discussions and question and answer sessions.
1. (a) Find the slope of the curve \( y = x^2 - 3x - 5 \) at the point \( P(2, -7) \) by finding the limit of the secant slopes through point \( P \).

(b) Find an equation of the tangent line to the curve at \( P(2, -7) \).

(a) The slope of the curve at \( P(2, -7) \) is \( \boxed{ } \). (Simplify your answer.)

(b) The equation of the tangent line to the curve at \( P(2, -7) \) is \( y = \boxed{ } \).
2. The accompanying graph shows the total distance traveled by a bicyclist after t hours.

Using the graph, answer parts (a) through (c).

(a) Which of the following is the bicyclist's average speed, in mph, over the time interval [0, 1]?

- **A.** 8 mph
- **B.** –8 mph
- **C.** 58 mph
- **D.** –58 mph

Which of the following is the bicyclist's average speed, in mph, over the time interval [1, 2.5]?

- **A.** 5.3 mph
- **B.** –5.3 mph
- **C.** –30 mph
- **D.** 30 mph

Which of the following is the bicyclist's average speed, in mph, over the time interval [2.5, 3.5]?

- **A.** –6 mph
- **B.** 56 mph
- **C.** 6 mph
- **D.** –56 mph

(b) Which of the following is the bicyclist's instantaneous speed, in mph, at t = \( \frac{1}{2} \) hr?

- **A.** 8 mph
- **B.** 58 mph
- **C.** –58 mph
- **D.** –8 mph

Which of the following is the bicyclist's instantaneous speed, in mph, at t = 2 hrs?

- **A.** 0 mph
- **B.** 1 mph
- **C.** –1 mph
- **D.** 2 mph

Which of the following is the bicyclist's instantaneous speed, in mph, at t = 3 hrs?

- **A.** 7 mph
- **B.** –32 mph
- **C.** –18 mph
- **D.** 32 mph

(c) Which of the following choices gives the maximum speed, in mph, and the time at which it
2. (cont.)

The maximum speed of the bicyclist is 16 mph and it occurs when \( t = 1 \) hr.

\( \text{B.} \) The maximum speed of the bicyclist is 16 mph and it occurs when \( t = 3.5 \) hrs.

\( \text{C.} \) The maximum speed of the bicyclist is 41 mph and it occurs when \( t = 3.5 \) hrs.

\( \text{D.} \) The maximum speed of the bicyclist is 41 mph and it occurs when \( t = 1 \) hr.

3. Find \( \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \).

\[ \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = \square \]

(Type an integer or a simplified fraction.)

4. Find \( \lim_{x \to -12} \frac{11 - \sqrt{x^2 - 23}}{x + 12} \).

\[ \lim_{x \to -12} \frac{11 - \sqrt{x^2 - 23}}{x + 12} = \square \]

(Type an integer or a simplified fraction.)

5. Find the limit.

\[ \lim_{x \to -4\pi} \sqrt{x + 15} \cos (x + 4\pi) \]

\[ \lim_{x \to -4\pi} \sqrt{x + 15} \cos (x + 4\pi) = \square \]

(Type an exact answer, using \( \pi \) as needed.)
6. Give an \(\varepsilon-\delta\) proof of \(\lim_{{x \to 4}} \left( \frac{x^2 - 16}{x - 4} \right) = 8\).

Let \(\varepsilon > 0\) be given.

- **A.** Let \(\delta = 4\varepsilon\). Then \(0 < |x - 4| < \delta \Rightarrow \left| \frac{x^2 - 16}{x - 4} - 8 \right| = \frac{1}{4} |x + 4 - 8| = \frac{1}{4} |x - 4| < \frac{1}{4} \delta = \varepsilon\).

- **B.** Let \(\delta = 2\varepsilon\). Then \(0 < |x - 4| < \delta \Rightarrow \left| \frac{x^2 - 16}{x - 4} - 8 \right| = \frac{1}{2} |x + 4 - 8| = \frac{1}{2} |x - 4| < \frac{1}{2} \delta = \varepsilon\).

- **C.** Let \(\delta = \varepsilon\). Then \(0 < |x - 4| < \delta \Rightarrow \left| \frac{x^2 - 16}{x - 4} - 8 \right| = |(x + 4) - 8| = |x - 4| < \delta = \varepsilon\).

- **D.** None of the above proofs is correct.

7. For the function graphed to the right, explain why \(\lim_{{x \to 2}} f(x) \neq 7\).

Choose the correct reason below.

- **A.** The limit of \(f(x)\) as \(x\) approaches 2 does not exist.
- **B.** The limit of \(f(x)\) as \(x\) approaches 2 is 5.
- **C.** The limit of \(f(x)\) as \(x\) approaches 2 is \(\frac{11}{2}\).
- **D.** The limit of \(f(x)\) as \(x\) approaches 2 is 4.
8. The figure given below shows the time-to-distance graph for an object accelerating from a standstill. (a) Estimate the slopes of secants PQ_1, PQ_2, PQ_3, and PQ_4, and arrange them in order in a table.

<table>
<thead>
<tr>
<th>PQ_1</th>
<th>PQ_2</th>
<th>PQ_3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. The graph below shows the profits of a small home business for the years of the decade of the 1990's. Find the average rate of change of profits for the interval specified and the rate at which profits were changing in the specified year. The average rate of change of profits for the interval year 6 through 8 is \$\square\ per year.

The rate at which profits were changing in year 5 of the decade is \$\square\ per year. (Round to the nearest thousand as needed.)
10. Graph the function \( y = -\frac{3}{2} \left( x - \frac{4}{x} \right)^{2/3} \). Then answer parts (a) through (c).

Choose the correct graph below.

\[ \text{A.} \quad \quad \text{B.} \quad \quad \text{C.} \quad \quad \text{D.} \]

(a) How does the graph behave as \( x \to 0^+ \)?

\[ \text{A.} \quad \text{The graph is not defined as } x \to 0^+. \]
\[ \text{B.} \quad \text{The graph approaches 0 as } x \to 0^+. \]
\[ \text{C.} \quad \text{The graph approaches } \infty \text{ as } x \to 0^+. \]
\[ \text{D.} \quad \text{The graph approaches } -\infty \text{ as } x \to 0^+. \]

(b) How does the graph behave as \( x \to \pm \infty \)?

\[ \text{A.} \quad \text{The graph is not defined as } x \to \pm \infty. \]
\[ \text{B.} \quad \text{The graph approaches 0 as } x \to \pm \infty. \]
\[ \text{C.} \quad \text{The graph approaches } \infty \text{ as } x \to \pm \infty. \]
\[ \text{D.} \quad \text{The graph approaches } -\infty \text{ as } x \to \pm \infty. \]

(c) How does the graph behave near \( x = 2 \) and \( x = -2 \)? Use the TRACE function on the graphing utility.

\[ \text{A.} \quad \text{The graph is not defined near } x = 2 \text{ and } x = -2. \]
\[ \text{B.} \quad \text{The graph approaches } -\infty \text{ near } x = 2 \text{ and } x = -2. \]
\[ \text{C.} \quad \text{The graph approaches 0 near } x = 2 \text{ and } x = -2. \]
\[ \text{D.} \quad \text{The graph approaches } \infty \text{ near } x = 2 \text{ and } x = -2. \]

11. An object is dropped from the top of a cliff 660 meters high. Its height above the ground \( t \) seconds after it is dropped is \( 660 - 4.9t^2 \). Determine its speed 10 seconds after it is dropped.

The speed of the object 10 seconds after it is dropped is \( \square \) m/sec.

(Simplify your answer.)
12. What is the rate of change of the volume of a ball \( V = \frac{4}{3} \pi r^3 \) with respect to the radius when the radius is \( r = 5 \)?

The volume changes at a rate of □.

(Type an exact answer, using \( \pi \) as needed.)
13. (a) Find the derivative $f'(x)$ of the function $f(x) = \frac{x^3}{11}$.
(b) Graph $f(x)$ and $f'(x)$ side by side using separate sets of coordinate axes.
(c) For what values of $x$, if any, is $f'$ positive? Zero? Negative?
(d) Over what intervals of $x$-values, if any, does the function $y = f(x)$ increase as $x$ increases? Decrease as $x$ increases? How is this related to the findings in part (c)?

(a) $f'(x) =$

(b) Choose the correct answer below.

![Graphs A, B, C, D]

(c) Determine the $x$-values for which $f'(x) > 0$, $f'(x) = 0$, and $f'(x) < 0$. Choose the correct answer below.

A. $f'(x) > 0$ for $x > 0$; $f'(x) = 0$ for $x < 0$; and $f'(x) < 0$ for $x = 0$.
B. $f'(x) < 0$ for $x < 0$; $f'(x) = 0$ for $x > 0$; and $f'(x) > 0$ for $x = 0$.
C. $f'(x) < 0$ for $x < 0$ and $x > 0$; $f'(x) = 0$ for $x = 0$; and it is not positive for any value of $x$.
D. $f'(x) > 0$ for $x < 0$ and $x > 0$; $f'(x) = 0$ for $x = 0$; and it is not negative for any value of $x$.

(d) Which of the following statements is correct about the increasing and decreasing nature of the function $y = f(x)$?

A. The function $y = f(x)$ decreases on the interval $(-\infty,0) \cup (0,\infty)$ as $x$ increases; and for every $x$ in this interval, $f'(x) > 0$.
B. The function $y = f(x)$ decreases on the interval $(-\infty,0) \cup (0,\infty)$ as $x$ increases; and for every $x$ in this interval, $f'(x) < 0$.
C. The function $y = f(x)$ increases on the interval $(-\infty,0) \cup (0,\infty)$ as $x$ increases; and for every $x$ in this interval, $f'(x) > 0$.
D. The function $y = f(x)$ increases on the interval $(-\infty,0) \cup (0,\infty)$ as $x$ increases; and for every $x$ in this interval, $f'(x) < 0$. 
14. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket coasts upward for a while and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands. The figure here shows the velocity data from the flight of the model rocket. Use the data to complete parts (a) through (g).

(a) How fast was the rocket climbing when the engine stopped?

[ ] ft/sec
(Round to the nearest integer as needed.)

(b) For how many seconds did the engine burn?

[ ] sec
(Type an integer or a decimal. Round to one decimal place as needed.)

(c) When did the rocket reach its highest point?

[ ] sec
(Type an integer or a decimal. Round to one decimal place as needed.)

What was the rocket’s velocity when it reached its highest point?

[ ] ft/sec
(Round to the nearest integer as needed.)

(d) When did the parachute pop out?

[ ] sec
(Type an integer or a decimal. Round to one decimal place as needed.)

How fast was the rocket falling when the parachute popped out?

[ ] ft/sec
(Round to the nearest integer as needed.)

(e) How long did the rocket fall before the parachute opened?

[ ] sec
(Type an integer or a decimal. Round to one decimal place as needed.)
14. (cont.)

(f) When was the rocket's acceleration the greatest?

The greatest acceleration happened $\Box$ sec after launch.
(Type an integer or a decimal. Round to one decimal place as needed.)

(g) When was the acceleration constant?

Acceleration was constant between $\Box$ and $\Box$ sec.
(Type integers or decimals. Round to one decimal place as needed.)

What was the acceleration then?

$\Box$ ft/sec$^2$
(Round to the nearest integer as needed.)

15. An object is dropped from a tower, 158 ft above the ground. The object's height above ground $t$ sec into the fall is $s = 158 - 16t^2$.

a. What is the object's velocity, speed, and acceleration at time $t$?

b. About how long does it take the object to hit the ground?

c. What is the object's velocity at the moment of impact?

The object's velocity at time $t$ is $\Box$.

The object's speed at time $t$ is $\Box\frac{ft}{sec}$.

The object's acceleration at time $t$ is $\Box\frac{ft}{sec^2}$.
(Simplify your answer.)

It takes $\Box$ sec for the object to hit the ground.
(Round to the nearest tenth.)

The object's velocity at the moment of impact is $\Box\frac{ft}{sec}$.
(Round to the nearest tenth.)
1. \[ \frac{1}{x - 9} \]

2. A
   B
   C
   A
   A
   A
   A

3. \[ \frac{1}{10} \]

4. \[ \frac{12}{11} \]

5. \[ \sqrt{15 - 4\pi} \]

6. C

7. A

8. 43
   46
   49
   50

9. 7000
   5000

10. B
    D
    D
    C

11. 98

12. 100\pi
13. $\frac{3x^2}{11}$
   B
   D
   C

14. 190
    2.0
    8.0
    0
    10.4
    75
    2.4
    2.0
    2.0
    10.4
    -32

15. $-32t$
    32t
    -32
    3.1
    -99.2