Math 1316 Trigonometry

3.2 Applications of Radian Measure
(Solutions are at the end)

**Arc Length**
The length $s$ of the arc intercepted on a circle of radius $r$ by central angle of measure $\theta$ radians is given by $s = r\theta$

1. Find the length of the arc

2. Find the radius of the circle

3. Find the measure of the central angle in degrees

4. Find the length of the arc intercepted by a central angle of $210^\circ$ and radius 12.4 feet.

Find the distance between each pair of cities. Assume the Earth is a sphere of radius 4000 miles and that the cities are on the same meridian (one city is due north of the other).

5. San Francisco $37^\circ46'39''$ N latitude and Seattle $47^\circ36'32''$ N latitude.

7. What is the difference in latitudes of two cities, one of which is 1850 miles due north of the other?

8. Find the radius of the larger wheel if the smaller wheel of radius 14.6” rotates 120° when the larger wheel rotates 60°.

9. Find the radius of the pulley if a rotation of 51.6° raises the weight 11.4 cm.

![Diagram of two wheels and a pulley](image)

Area of a Sector
The area $A$ of a sector of a circle of radius $r$ and central angle $\theta$ measured in radians is given by

$$A = \frac{1}{2} r^2 \theta$$

10. Find the area of the sector of the circle with radius $r = 30$ ft and central angle $81°$. 


Solutions:

1. $S = r \theta$
   
   $S = 7 \left( \frac{2\pi}{3} \right) = 14.66 \text{ inches}$

2. $S = r \theta$
   
   $15 = r \left( \frac{\pi}{12} \right)$
   
   $r = 15 \left( \frac{12}{\pi} \right) = 57.3 \text{ cm}$

3. $S = r \theta$
   
   $29.32 = 8\theta$
   
   $\theta = \frac{29.32}{8} \text{ radians or } \theta = \frac{29.32 \left( \frac{180^\circ}{\pi} \right)}{8} = 209.989^\circ$

4. The angle needs to be converted to radian measure $\theta = 210^\circ = 210^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{7\pi}{6}$
   
   $s = 12.4 \left( \frac{7\pi}{6} \right) = 45.4 \text{ feet}$

5. $\theta = 47^\circ 36' 32'' - 37^\circ 49' 53'' = 9.831389^\circ = 9.831389 \left( \frac{\pi}{180^\circ} \right) = 0.17159 \text{ radians}$
   
   $S = r \theta = 4000(0.17159) = 686.4 \text{ miles apart}$

6. $\theta = 26^\circ 10' + 31^\circ 47' = 57.95^\circ = 57.95 \left( \frac{\pi}{180^\circ} \right) = 1.011418 \text{ radians}$
   
   $S = r \theta = 4000(1.011418) = 4045.67 \text{ miles apart}$

7. $S = r \theta$
   
   $1850 = (4000) \theta$
   
   $\theta = \frac{1850}{4000} = 0.4625 \text{ radians} = 0.4625 \left( \frac{180^\circ}{\pi} \right) = 26.5^\circ$

8. $S_1 = S_2$
   
   $r_1 \theta_1 = r_2 \theta_2$ (must convert degrees to radian measure)
   
   $120^\circ = 120^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{2\pi}{3}$
   
   $60^\circ = 60^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{\pi}{3}$

   $14.6 \left( \frac{2\pi}{3} \right) = r_2 \left( \frac{\pi}{3} \right)$
   
   $r_2 = 29.2 \text{ inches}$

9. $S = r \theta$ (convert the angle to radian measure)
   
   $\theta = 51.6^\circ = 51.6^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{51.6\pi}{180} = 0.9006$

   $11.4 = r(0.9006)$
   
   $r = 12.7 \text{ cm}$

10. $\theta = 81^\circ = 81^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{81\pi}{180} = \frac{9\pi}{2}$

   $A = \frac{1}{2} r^2 \theta = \frac{1}{2} 30^2 \cdot \frac{81\pi}{180} = 636.2 \text{ ft}^2$