3.1 Radian Measure

Definition: An angle with its vertex at the center of circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of 1 radian.

The first angle has measure 1 radian since the arc length is $s = 1 \cdot r$ the second angle has measure 2 radians since the arc length is $s = 2 \cdot r$.

The measure of this angle is $2\pi$ radians since the arc length is $s = 2\pi \cdot r$

This result shows us that $360^\circ = 2\pi$ radians.

**CONVERTING BETWEEN DEGREES AND RADIANS**

1. To convert from degrees to radians multiply degrees by $\frac{\pi}{180^\circ}$ and simplify
2. To convert from radians to degrees multiply radians by $\frac{180^\circ}{\pi}$ and simplify
Convert each radian measure to degrees

1. \( \frac{7\pi}{4} \)  
2. \( \frac{-8\pi}{5} \)  
3. \( \frac{17\pi}{20} \)  
4. 9.84763

Solution:
1. \( \frac{7\pi}{4} = \frac{7\pi}{4} \cdot \left( \frac{180^\circ}{\pi} \right) = 315^\circ \)
2. \( \frac{-8\pi}{5} = \frac{-8\pi}{5} \cdot \left( \frac{180^\circ}{\pi} \right) = -288^\circ \)
3. \( \frac{17\pi}{20} = \frac{17\pi}{20} \cdot \left( \frac{180^\circ}{\pi} \right) = 153^\circ \)
4. 9.84763 = 9.84763 \cdot \left( \frac{180^\circ}{\pi} \right) = 564.22764^\circ 

Convert each degree measure to radian measure

5. 23\(^\circ\)  
6. 139\(^\circ\)10’  
7. 23.0143\(^\circ\)  
8. 122\(^\circ\)17’47”

Solution:
5. \( 23^\circ = 23^\circ \cdot \left( \frac{\pi}{180^\circ} \right) = \frac{23\pi}{180} \)
6. 139\(^\circ\)10’ = 139\(^\circ\)10’ \cdot \frac{\pi}{180^\circ} = 2.43

Trigonometric function values for angles measured in radians can be found by first converting radian measure to degrees.

Find the exact value of each expression without using a calculator.

9. \( \sin \frac{\pi}{3} \)  
10. \( \tan \frac{5\pi}{3} \)  
11. \( \cos \frac{8\pi}{3} \)  
12. \( \sin \frac{7\pi}{6} \)  
13. \( \cot \frac{13\pi}{3} \)

Solution:
9. \( \sin \frac{\pi}{3} = \sin \left( \frac{\pi}{3} \cdot \frac{180^\circ}{\pi} \right) = \sin 60^\circ = \frac{\sqrt{3}}{2} \)

10. \( \tan \frac{5\pi}{3} = \tan \left( \frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} \right) = \tan 300^\circ = -\sqrt{3} \)

11. \( \cos \frac{8\pi}{3} = \cos \left( \frac{8\pi}{3} \cdot \frac{180^\circ}{\pi} \right) = \cos(-120^\circ) = -\frac{1}{2} \)

12. \( \sin \frac{7\pi}{6} = \sin \left( \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} \right) = \sin(-210^\circ) = -\frac{1}{2} \)

13. \( \cot \frac{13\pi}{3} = \cot \left( \frac{13\pi}{3} \cdot \frac{180^\circ}{\pi} \right) = \cot 780^\circ = \frac{\sqrt{3}}{3} \)