2.5 Equations of Lines

Point-Slope Form
The equation of the line with slope \( m \) passing through the point \( (x_1, y_1) \) is

\[
y - y_1 = m(x - x_1)
\]

Slope-Intercept Form
The equation of the line with slope \( m \) and \( y \)-intercept \( b \) is

\[
y = mx + b
\]

And an equation of line in **standard form** is \( Ax + By = C \)

Examples: Write the equation of each line in a) Standard Form

b) Slope-Intercept form, if possible

1. The line through \( (1,3), m = -2 \)
   Solution:
   \[
y - y_1 = m(x - x_1)
   \]
   \[
y - 3 = -2(x - 1)
   \]
   \[
y - 3 = -2x + 2
   \]
   Standard Form: \( 2x + y = 5 \)
   Slope-Intercept Form: \( y = -2x + 5 \)

2. The line through \( (-5,4), m = -\frac{3}{2} \)
   \[
y - y_1 = m(x - x_1)
   \]
   \[
y - 4 = -\frac{3}{2}(x - (-5))
   \]
   \[
   2(y - 4) = 2 \left[-\frac{3}{2}(x + 5)\right]
   \]
   \[
   2y - 8 = -3(x + 5)
   \]
   \[
   2y - 8 = -3x - 15
   \]
   Standard Form: \( 3x + 2y = -7 \)
   Slope Intercept Form: \( 2y = -3x - 7 \)
   \[
   y = -\frac{3}{2}x - \frac{7}{2}
   \]

3. The line through \( (8, -1) \) and \( (4,3) \)
   \[
m = \frac{y_2-y_1}{x_2-x_1} = \frac{3 - (-1)}{4 - 8} = \frac{4}{-4} = -1
   \]
\[ y - y_1 = m(x - x_1) \]
\[ y - 1 = -1(x - 8) \]
\[ y + 1 = -x + 8 \]
Standard Form: \( x + y = 7 \)
Slope-Intercept Form: \( y = -x + 7 \)

4. x-intercept -2, y-intercept 4
\((-2,0)\) and \((0,4)\)
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2 \]
\[ y - 4 = 2(x - 0) \]
\[ y - 4 = 2x \]
Standard Form: \( 2x - y = -4 \)
Slope Intercept Form: \( y = 2x + 4 \)

**Vertical Line**
The equation of the vertical line (\( m \) is undefined) passing through the point \((a, b)\) is
\[ x = a \]

**Horizontal Line**
The equation of the horizontal line (\( m = 0 \)) passing through the point \((a, b)\) is
\[ y = b \]

5. horizontal, through \((-8, -2)\)
\[ y = -2 \]

6. through \((5,1)\), undefined slope
\[ x = 5 \]

Two nonvertical lines are
1) Parallel if and only if their slopes are equal.
2) Perpendicular if and only if their slopes are negative reciprocals.
7. Through \((-1,4)\) parallel to \(2x + 3y = -6\)

\(2x + 3y = -6\)
\[3y = -2x + 6\]
\[y = -\frac{2}{3}x + 2\]
\[y - y_1 = m(x - x_1)\]
\[y - 4 = -\frac{2}{3}(x - 1)\]

\[3(y - 4) = 3\left[-\frac{2}{3}(x - 1)\right]\]
\[3y - 12 = -2(x + 1)\]
\[3y - 12 = -2x - 2\]

Standard Form: \(2x + 3y = 10\)
Slope Intercept Form: \(3y = -2x + 10\)
\[y = -\frac{2}{3}x + \frac{10}{3}\]

8. Through \((3,5)\) perpendicular to \(3x - 5y = 15\)

We first find the slope by rewriting in slope-intercept form.
\[3x - 5y = 15\]
\[-5y = -3x + 15\]
\[y = \frac{3}{5}x - 3\]

The slope of a line perpendicular to this line is \(m_\perp = -\frac{5}{3}\)
\[y - y_1 = m(x - x_1)\]
\[y - 5 = -\frac{5}{3}(x - 3)\]

\[3(y - 5) = 3\left[-\frac{5}{3}(x - 3)\right]\]
\[3(y - 5) = -5(x - 3)\]
\[3y - 15 = -5x + 15\]

Standard Form: \(5x + 3y = 30\)
\[3y = -5x + 30\]
Slope-Intercept Form: \(y = -\frac{5}{3}x + 10\)

9. Through \((-2,7)\) perpendicular to \(y = 5\)

Solution: The graph of the line \(y = 5\) is a horizontal line, therefore a line perpendicular to this line is a vertical line.

The Equation is \(x = -2\)