The set of all possible outcomes for an experiment is the **sample space** for that experiment. An **event** is a subset of a sample space. An event with only one possible outcome is a **simple event**. If an event equals the sample space, then the event is a **certain event**.

**Examples:** Write sample spaces for the following experiments.

1. A day in March is selected for a 5K running event.
   Solution: \( S = \{1,2,3,4,\ldots,31\} \)

2. The total number of possible points earned on an 80 point test.
   Solution: \( S = \{0,1,2,3,4,\ldots,80\} \)

3. A die is rolled.
   Solution: \( S = \{1,2,3,4,5,6\} \)

4. A coin is tossed and a die is rolled
   Solution:
   \[
   S = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}
   \]

5. Write the sample space for the number of boys and the number of girls in a family with exactly three children. Are the outcomes equally likely?
   Solution: \{3 boys, 2 boys a girl, 1 boy 2 girls, 3 girls\}. The outcomes are not equally likely, since the event of having three boys has only one possible outcome however the event of having 2 boys and one girl is \{bbg,bgb,gbb\} which has three possible outcomes.

6. Give an example of a simple event.
   Solution: From example 5, the event of having three boys is a simple event since it has only one possible outcome.

7. Give an example of a certain event.
   Solution: Two dice are rolled and the total sum is less than or equal to 12 is an example of a certain event.

In examples 8 and 9, write out an equally likely sample space, and then write the indicated events in set notation.

8. A marble is drawn at random from a bowl containing 3 yellow, 4 white, and 8 blue marbles.
   a) A yellow marble is drawn
   b) A white marble is drawn
   c) A blue marble is drawn
   d) A black marble is drawn
   Solution: The sample space is \( S = \{y_1, y_2, y_3, w_1, w_2, w_3, w_4, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\} \)
   
   a) \( \{y_1, y_2, y_3\} \)
   b) \( \{w_1, w_2, w_3, w_4\} \)
   c) \( \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\} \)
   d) \( \emptyset \)
9. A collection of shoes contains flats, 2” heels, and 3” heels and the shoes are either light beige, dark beige, or black.
   a) The shoe selected has a heel and is black
   b) The shoe selected has no heel and is beige
   c) The shoe selected has a heel and is beige
Solution: The sample space is: 
\[ S = \{(l,f), (d,f), (b,f), (l,2), (d,2), (b,2), (l,3), (d,3), (b,3)\} \]
a) \( \{(b,2), (b,3)\} \)
b) \( \{(l,f), (d,f)\} \)
c) \( \{(l,2), (d,2), (l,3), (d,3)\} \)

10. Six people live in a dorm suite. Two are to be selected to go to the campus café to pickup a pizza. Of course, no one wants to go, so the six names (Connie, Casey, Lindsey, Jackie, Taisa, and Lisa) are placed in a hat. After the hat is shaken, two names are selected.
   a) Both names selected begin with the letter L
   b) The first name selected begins with L and the second name begins with J or C
   c) The first name selected begins with the letter C
Solution:
   a) \( \{(Lindsey, Lisa)\} \)
   b) \( \{(Lindsey, Connie),(Lisa, Connie),(Lindsey, Casey),(Lisa, Casey),(Lindsey, Jackie),(Lisa, Jackie,}\} \)
   c) \( \{(Connie, Casey), (Connie, Lindsey), (Connie, Jackie), (Connie, Taisa), \{Connie, Lisa), (Casey, Lindsey), \{Casey, Jackie), (Casey, Taisa), (Casey, Lisa), (Casey, Connie)\} \)

### Basic Probability Principle

Let S be a sample space of equally likely outcomes, and let event E be a subset of S.
Then the probability that event E occurs is

\[ P(E) = \frac{n(E)}{n(S)} \]

11. A single fair die is rolled. Find the probabilities of the following:
   a) Getting a 5
   b) Getting a number less than 4
   c) Getting a 2 or a 5
   d) Getting any number except 3
Sample space: \( S = \{1,2,3,4,5,6\} \)
   a) \( E = \{5\} \) so \( P(E) = \frac{n(E)}{n(S)} = \frac{1}{6} \)
   b) \( E = \{1,2,3\} \) so \( P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} \)
   c) \( E = \{2,5\} \) so \( P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3} \)
   d) \( E = \{1,2,4,5,6\} \) so \( P(E) = \frac{n(E)}{n(S)} = \frac{5}{6} \)
12. A card is drawn from a well-shuffled deck of 52 cards. Find the probabilities of drawing each of the following.
   a) An 8
   b) A red card
   c) A red 10
   d) A 7 or a king
   e) A red 6 or a black 9

   Solution: a) \( E = \{(8, D), (8, H), (8, C), (8, S)\} \) \( P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13} \)

   b) There are 13 hearts and 13 diamonds \( P(E) = \frac{n(E)}{n(S)} = \frac{26}{52} = \frac{1}{2} \)

   c) There are 2 red 10's so \( P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26} \)

   d) There are four 7's and four kings so \( P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13} \)

   e) There are two red 6's and two black 9's so \( P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13} \)