5.2 Compound Interest
(Solutions are at the end)

Simple interest is normally used for loans or investments of a year or less. For longer terms, compound interest is used. With compound interest, interest is charged (or paid) on interest as well as principal.

### Compound Interest

If $P$ dollars is invested at an annual interest $r$ compounded $n$ times per year, then the future value $A$ after $t$ years is

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

In examples 1-3, find the future value (or compounded amount) for each investment

1. $8000 at 7\% compounded annually for 12 years.
2. $6500 at 5.5\% compounded quarterly for 6 years.
3. $45,500 at 2.5\% compounded daily for 30 years.
4. Find the amount of interest earned by investing $27,630.35 at 4.4\% compounded quarterly for 3.7 years.
5. Find the interest rate (with annual compounding) if an investment of $9000 grows to $17,118 in 16 years.

### Continuous Compounding

If $P$ dollars is invested at an annual interest $r$ compounded continuously, then the future value $A$ after $t$ years is

$$A = Pe^{rt}$$

6. Find the future value if $18,500 is invested at 6.5\% compounded continuously for 20 years.
7. Find the interest rate (with continuous compounding) if an investment of $8500 grows to $17,000 in 10 years.

If 1$ is deposited at 4\% compounded quarterly, a calculator can be used to find that at the end of one year the compounded amount is $1.0406, an increase of 4.06\% over the original $1. The actual increase of 4.06\% in the money is higher than the stated increase of 4\%. The stated 4\% is called the nominal rate of interest and the actual increase of 4.06\% is called the effective rate.
Effective Rate

The effective rate corresponding to a stated rate of interest \( r \) compounded \( n \)-times per year is

\[
\begin{align*}
\text{Effective Rate} & = \left( 1 + \frac{r}{n} \right)^n - 1
\end{align*}
\]

8. Find the effective rate of an investment at 5.2% compounded semiannually.

9. A company agrees to pay $2.9 million in 5-years to settle a lawsuit. How much must it invest now in an account paying 8% compounded monthly to have that amount when it is due?

10. The Flagstar Bank in Michigan offered a 5-year certificate of deposit at 4.38% interest compounded quarterly. On the same day, Principal Bank offered a 5-year at 4.37% interest compounded monthly. Find the APY (effective rate) for each CD. Which bank offered a higher APY?

11. Scott Silva borrowed $5200 from his friend Joe Vetere to buy computer equipment. He repaid the loan 10 months later with simple interest at 7%. Vetere then invested the proceeds in a 5-year CD paying 6.3% compounded quarterly. How much will he have at the end of 5-years?

12. You decide to invest $16,000 in a money market fund that guarantees a 5.5% annual interest rate compounded monthly for 7 years. A one time fee of $30 is charged to set up the account. In addition, there is an annual charge of 1.25% of the balance in the account at the end of each year.
   a) How much is in the account at the end of the first year?
   b) How much is in the account at the end of the seventh year?

Solutions:

1. \[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]
   \[ = 8000 \left( 1 + \frac{0.07}{1} \right)^{12} = 18,017.53 \]

2. \[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]
   \[ = 6500 \left( 1 + \frac{0.055}{4} \right)^{24} = 9020.99 \]

3. \[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]
   \[ = 45,500 \left( 1 + \frac{0.025}{365} \right)^{10950} = 96,321.03 \]

4. \[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]
   \[ = 27,630.35 \left( 1 + \frac{0.044}{4} \right)^{4(3.7)} = 32,486.54 \]

Interest = $32,486.54 − $27,630.35 = $4856.19
5. \[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ 17,118 = 9000(1 + r)^{16} \]
\[ \frac{17,118}{9000} = (1 + r)^{16} \]
\[ \left(\frac{17,118}{9000}\right)^{\frac{1}{16}} = 1 + r \]
\[ \frac{17,118}{9000} - 1 = r \]
\[ r = 4.1\% \]

6. \[ A = Pe^{rt} \]
\[ A = 18,500e^{(0.065 \cdot 20)} = \$67,882 \]

7. \[ A = Pe^{rt} \]
\[ 17,000 = 8500e^{10r} \]
\[ 2 = e^{10r} \]
\[ \ln 2 = \ln e^{10r} \]
\[ \ln 2 = 10r \]
\[ r = \frac{\ln 2}{10} = 6.9\% \]

8. \[ r_E = \left(1 + \frac{r}{n}\right)^n - 1 \]
\[ r_E = \left(1 + \frac{0.052}{2}\right)^2 - 1 = 5.27\% \]

9. \[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ 2.9 = P \left(1 + \frac{0.08}{12}\right)^{60} \]
\[ P = \frac{2.9}{\left(1 + \frac{0.08}{12}\right)^{60}} = \$1.95 \text{ million} \]

10. \[ r_E = \left(1 + \frac{r}{n}\right)^n - 1 \]
\[ r_E = \left(1 + \frac{0.0438}{4}\right)^4 - 1 \]
\[ r_E = 4.452\% \]
\[ r_E = \left(1 + \frac{0.0437}{12}\right)^{12} - 1 \]
\[ r_E = 4.459\% \]
Principal Bank offers the greater APY
11. Simple Interest: $P = 5200 \left(1 + 0.07 \left(\frac{10}{12}\right)\right) = \$5503.33$

Compound Interest: $P = 5503.33 \left(1 + \frac{0.063}{4}\right)^{20} = \$7522.50$ at the end of 5-years

12. $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$A = 15970 \left(1 + \frac{0.55}{12}\right)^{12} = \$16,870.83$

Now deduct 1.25% gives $A = 16,870.83 - 16,870.83(0.0125) = \$16,659.95$

The amount after the first year is $16,659.95$. We proceed in the same manner six more times to find the amount after the seventh year.

$A = 16,659.95 \left(1 + \frac{0.55}{12}\right)^{12} = 17,599.70$. Now deduct 1.25% to get $17,379.70$

$A = 17,379.70 \left(1 + \frac{0.55}{12}\right)^{12} = 18,360.05$. Now deduct 1.25% to get $18,130.55$

$A = 18,130.55 \left(1 + \frac{0.55}{12}\right)^{12} = 19,153.26$. Now deduct 1.25% to get $18,913.84$

$A = 18,913.84 \left(1 + \frac{0.55}{12}\right)^{12} = 19,980.73$. Now deduct 1.25% to get $19,730.97$

$A = 19,730.97 \left(1 + \frac{0.55}{12}\right)^{12} = 20,843.95$. Now deduct 1.25% to get $20,583.40$

$A = 20,583.40 \left(1 + \frac{0.55}{12}\right)^{12} = 21,744.47$. Now deduct 1.25% to get $21,472.67$

At the end of the 7th year there will be $21,472.67 in the account.