5.1 Simple Interest

1. The simple interest \( I \) on \( P \) dollars at a rate of interest \( r \) per year for \( t \) years is
   \[ I = Prt \]

2. The future value \( A \) of \( P \) dollars for \( t \) years at a simple rate of interest \( r \) per year is
   \[ A = P(1 + rt) \]

3. The present value \( P \) of a future amount of \( A \) dollars at a simple rate of interest \( r \) for \( t \) years is
   \[ P = \frac{A}{1 + rt} \]

In problems 1 - 3, calculate the simple interest.

1. $1875 at 5.3% for 7 months
   Solution: \( I = Prt \)
   \[ = 1875(0.053)(\frac{7}{12}) \]
   \[ = 57.97 \]

2. $2830 at 8.9% for 125 days
   Solution: \( I = Prt \)
   \[ = 2830(0.089)(\frac{125}{365}) \]
   \[ = 86.26 \]

3. $39,086 at 9.4%; loan made on September 12 and due July 30.
   Solution \( I = Prt \)
   \[ = 39,086(0.094)(\frac{321}{365}) \]
   \[ = 3231.18 \]

4. Find the present value given the future amount $29,764 if the loan is for 310 days earning 7.2% interest.
   Solution: \( P = \frac{A}{1 + rt} \)
   \[ = \frac{29764}{1 + 0.072(\frac{310}{365})} \]
   \[ = 28,048.80 \]

The loans thus far are called **simple interest notes**, where interest on the face value of the loan is added to the loan and paid at maturity. Another common type of a note, is called a **simple discount note**, has the interest deducted in advance from the amount of a loan before giving the balance to the borrower.

**Illustration**: A Consider a loan for $3000 at 6% for 1-year.

<table>
<thead>
<tr>
<th>Interest on the note</th>
<th>Simple Interest Note</th>
<th>Bank Discount Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrower receives</td>
<td>$3000</td>
<td>$3000 - $185 = $2820</td>
</tr>
<tr>
<td>Borrower pays back</td>
<td>$3180</td>
<td>$3000</td>
</tr>
</tbody>
</table>

The money that is deducted is called the **discount** and the money actually received by the borrower is called the **proceeds**. From the illustration above; the discount is $180 and the proceeds is $2820.
5. Find the proceeds for a loan of $50,900 for 238 days at a discount rate of 8.2%.
Solution: \[ D = Prt = 50,900(0.082) \left( \frac{238}{365} \right) = $2721.55 \]
Proceeds = $50,900 − $2721.55 = $48,178.45

In Example #5, the borrower was charged a discount of 8.2%. However, 8.2% is not the interest paid, since 8.2% applies to $50,900, while the borrower actually only received $48,178.45. In the next example, we find the rate of interest actually paid by the borrower.

6. Find the interest rate to the nearest tenth on the proceeds for the simple interest note of $43,000 for 4 months if the discount rate is 9%.
Solution: \[ D = Prt = 43,000(0.09) \left( \frac{4}{12} \right) = $1290 \]
Proceeds = $43,000 − $1290 = $41,710
\[ I = Prt \]
\[ 1290 = 41,710(r) \left( \frac{4}{12} \right) \]
\[ r = \frac{1290}{41,710 \left( \frac{4}{12} \right)} = 9.28\% \]

7. Mike Branson invested his summer earnings of $3000 in a savings account for college. The account pays 2.5% interest. How much will this amount to in 9 months?
Solution: \[ A = P(1 + rt) = 3000 \left( 1 + 0.025 \cdot \frac{9}{12} \right) = $3056.25 \]

8. A $100,000 certificate of deposit held for 60 days is worth $101,133.33. To the nearest tenth of a percent, what interest rate was earned?
Solution: \[ A = P(1 + rt) \]
\[ \frac{101,133.33}{100,000} = 1 + r \cdot \frac{60}{365} \]
\[ \frac{101,133.33}{100,000} - 1 = r \cdot \frac{60}{365} \]
\[ \frac{101,133.33}{100,000} - 1 = r \]
\[ r = 6.9\% \]

9. A firm has ordered 7 new computers at a cost of $5104 each. The machines will not be delivered for 7 months. What amount could the firm deposit in an account paying 6.42% to have enough to pay for the machines?
Solution: The firm needs a total of 7(5104) = $35,728 in 7 months, this is the future value of an investment, we need the present value:
\[ P = \frac{A}{1 + rt} = \frac{35,728}{1 + 0.0642 \left( \frac{7}{12} \right)} = $34,438.29 \]
10. Johns Mathews signs a $4200 note at the bank, which charges a 12.2% discount rate. Find the net proceeds if the note is for 10 months. Find the actual interest rate (to the nearest hundredth) charged by the bank.

Solution: 
\[ D = Prt = 4200 \times (0.122) \times \left( \frac{10}{12} \right) = 427 \]
Proceeds = $4200 - $427 = $3773

Interest rate: 
\[ I = Prt \]
\[ 427 = 3773 \times r \times \frac{10}{12} \]
\[ r = \frac{427}{3773 \times \frac{10}{12}} = 13.58\% \]

10. A plumber loans $13,500 to a building contractor. The note is due in 9 months, with interest of 9%. Three months after the note is signed, the plumber discounts it at the bank. The bank charges a 10.1% discount rate. How much will the plumber receive? Will it be enough to pay a bill for $13,582?

Solution: The plumber will receive 
\[ A = 13,500 \left( 1 + 0.09 \left( \frac{9}{12} \right) \right) = 14,411.25 \]
from the contractor at the end of 9-months. Three months after the note is signed, the plumber discounts this amount at the bank for the remaining 6-months. The proceeds from the bank at three months is
\[ P = 14,411.25 \times \left( 1 - 0.101 \left( \frac{6}{12} \right) \right) = 13,683.48 \]
This amount is enough to pay a bill for $13,582.