The Laws of Logs

Basic Ideas of Logs:

1. \( \log_2 8 \) means 2 to the “what exponent” would give you 8, in other words, \( 2^? = 8 \). So the exponent must be 3. Therefore, \( \log_2 8 = 3 \) since \( 2^3 = 8 \).

2. Log of base 10 (\( \log_{10} \)) is normally written without the 10; so for example, \( \log 100 = \log_{10} 100 \). This is asking 10 to the “what exponent” would give you 100 or \( 10^? = 100 \). The answer is 2. Therefore, \( \log 100 = 2 \) since \( 10^2 = 100 \).

3. Log of base e (\( \log_e \)) where e = 2.718…) is normally written without the e and ln is used in place of log. Thus, \( \log_e ? = \ln ? \). So \( \ln 7.389 \approx 2 \) since \( e^2 \approx 7.389 \).

Converting into Exponential Form:

Since \( \log_2 8 \) is asking \( 2^? = 8 \), we can let the ? be represented by a variable. Lets use x. Therefore, to convert \( \log_2 8 \) into exponential form we say \( 2^x = 8 \). Then you can solve for x.

Converting into Logarithmic Form:

This is just going backwards with the definition of log. If we started out with \( 2^x = 8 \), 2 is the base so it will be the base of our log, the exponent is what the log is equal to and 8 is what you are taking log of; thus, the logarithmic form is \( \log_2 8 = x \).

Operations on Logs:

1. \( \log_b x + \log_b y \leftrightarrow \log_b xy \)
   - For Example: \( \log_3 4 + \log_3 6 \leftrightarrow \log_3 4 \cdot 6 = \log_3 24 \)

2. \( \log_b x - \log_b y \leftrightarrow \log_b x/y \)
   - For Example: \( \log_3 10 + \log_3 5 \leftrightarrow \log_3 10 \div 5 = \log_3 2 \)

3. \( r \log_b x \leftrightarrow \log_b x^r \)
   - For Example: \( 2 \log_3 4 \leftrightarrow \log_3 4^2 = \log_3 16 \)

Other Rules on Logs:

1. \( \log_b b^x = x \) or \( \ln e^x = x \) (remember, \( \ln \) has understood base e, so \( \ln e^x = x \)) or \( \log 10^x = x \) (remember, \( \log \) has understood base 10, so \( \log_{10} 10^x = x \))

2. \( e^{\log_e x} = x \)
   - For Example: \( e^{\log_e 6} = 6 \)

3. \( b^{\log_b x} = x \)
   - For Example: \( 5^{\log_5 9} = 9 \)
4. \( \log_b x = \frac{\log x}{\log b} \)

- For Example: \( \log_3 4 \) is asking \( 3^? = 4 \), which is not a pretty answer. The calculator has a built-in function of \( \log_{10} \) and we can use the calculator to find \( \log_3 4 \) by using this rule.

\[
\log_3 x = \frac{\log_{10} 4}{\log_{10} 3} = 1.2618\
\]

Solving Equations Containing Logarithms or Exponents:

1. An equation that has a \( \log \) on one side and some expression on the other side can be written without the \( \log \).
   \[ \log_b x = y \quad \Leftrightarrow \quad b^y = x \]
   - For Example: \( \log_3 (x + 4) = 2 \quad \Leftrightarrow \quad 3^2 = x + 4 \)
     \[ 9 = x + 4 \quad \Leftrightarrow \quad 5 = x \]

2. An equation that has a \( \log \) with the same base on both sides can be written without the \( \log \).
   \[ \log_b x = \log_b y \quad \Leftrightarrow \quad x = y \]
   - For Example: \( \log_3 (3x + 1) = \log_3 (x + 9) \quad \Leftrightarrow \quad 3x + 1 = x + 9 \)
     \[ 2x = 8 \quad \Leftrightarrow \quad x = 4 \]

3. An equation that has only exponential expressions where the bases are the same can be written without the bases.
   \[ b^x = b^y \quad \Leftrightarrow \quad x = y \]
   - For Example: \( 5^{2x - 10} = 5^2 \quad \Leftrightarrow \quad 2x - 10 = 2 \)
     \[ 2x = 12 \quad \Leftrightarrow \quad x = 6 \]

4. An equation that contains exponential expressions but has different bases on each side of the equation, then the \( \log \) (or \( \ln \)) of each side of the equation is taken and then simplified.
   - For Example: \( 3^{2x - 10} = 5 \quad \Leftrightarrow \quad \log 3^{2x - 10} = \log 5 \)
     \[ (2x - 10)\log 3 = \log 5 \]
     \[ (2x - 10)\log 3 = \log 5 \]
     \[ \log 3 \quad \log 3 \]

   Note: This is just ONE way to solve this problem.
   \[ 2x - 10 = \frac{\log 5}{\log 3} \]
   \[ 2x = 10 + \frac{\log 5}{\log 3} \]
   \[ \frac{1}{3}2x = \frac{1}{3}(10 + \log 5) \]
   \[ \frac{1}{2}\log 3 \]
   \[ x = 5 + \frac{\log 5}{2\log 3} \quad \text{use the calculator} \]
   \[ x = 5.7324\ldots \]