Finding the Greatest Common Factor

Definition: The Greatest Common Factor (GCF) is the largest number/expression that ` divides into two or more expressions evenly.

For Example: For the numbers 18 and 27, 3 is a common factor, but 9 is the greatest common factor, since 9 is the largest number that divides into 18 and 27 evenly.

Finding the GCF: One approach to finding the GCF is looking at the prime factors that occurs the least (look for the smallest exponent) in each of the numbers or expressions that are involved. For instance, in the previous example, 18 and 27, factor each number into its prime factors.

$$\mathbf{18 = 3 \bullet 6 = 3 \bullet 3 \bullet 2 = 3^2 \bullet 2} \qquad \mathbf{27 = 3 \bullet 9 = 3 \bullet 3 \bullet 3 = 3^3}$$

The least exponent on the 3 is two and on the 2 is zero (since 27 does not have any factors of 2) so the GCF is $\mathbf{3^2 = 9}$**.**

Another example of finding the GCF of 90 and 120:

$$\mathbf{90 = 2 \bullet 3 \bullet 3 \bullet 5 = 2 \bullet 3^2 \bullet 5} \qquad \mathbf{120 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 5 = 2^3 \bullet 3 \bullet 5}$$

The least exponent of each factor is one so the GCF is $\mathbf{2 \bullet 3 \bullet 5 = 30}$**.**

Examples for Finding the GCF of Algebraic Expressions:

The same approach is used to find the GCF of algebraic expressions—factor into prime factors first.

Example: Find the GCF of $12x^2y^3w$ and $20x\ y^2$.

$$\mathbf{12x^2y^3w = 2^2 \bullet 3 \bullet} x^2 \bullet y^3 \bullet w \qquad \mathbf{20x\ y^2 = 2^2 \bullet 5 \bullet x \bullet y^2}$$

Choose the least exponent for each factor. So the GCF is $\mathbf{2^2 \bullet x \bullet y^2}$ (3, 5 or w did not occur in both expressions so they are not part of the GCF).

Example: Find the GCF of $3x^3 + 6x^2$ and $6x^2 – 24$

$$\mathbf{3x^3 + 6x^2 = 3x^2(x + 2) = 3 \bullet x^2 \bullet (x + 2)} \qquad \mathbf{6x^2 – 24 = 6(x^2 – 4) = 2 \bullet 3 \bullet (x + 2) \bullet (x - 2)}$$

The GCF is $\mathbf{3(x + 2)}$