Arithmetic of Fractions

Addition and Subtraction

If the denominators are the same, add or subtract the numerators, and leave the denominators the same.

Examples:
\[
\frac{3}{10} + \frac{4}{10} = \frac{7}{10} \quad \frac{4}{7} - \frac{3}{7} = \frac{1}{7}
\]

Note: You may need to reduce the answer even if the original parts are in lowest terms.

Example:
\[
\frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}
\]

If the denominators are different, first convert each fraction to the lowest common denominator (LCD), the least common multiple of the denominators. Divide the LCD by the denominator, then multiply the numerator and denominator by this factor to get a fraction with the LCD.

Example:
\[
\frac{1}{3} + \frac{2}{5}
\]

The LCD is 15, so convert as follows:
\[
\frac{15}{3} \div 3 = 5 \quad \frac{15}{5} \div 5 = 3
\]

\[
\frac{1}{3} \cdot \frac{5}{3} = \frac{5}{15} \quad \frac{2}{5} \cdot \frac{3}{5} = \frac{6}{15}
\]

Once the fractions are converted, add or subtract as usual (and reduce if necessary):
\[
\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}
\]

Important: Always multiply the numerator and denominator by the same number!

Example:
\[
\frac{1}{4} + \frac{2}{3} - \frac{1}{6}
\]

The LCD is 12, convert as follows:
\[
12 \div 4 = 3 \quad 12 \div 3 = 4 \quad 12 \div 6 = 2
\]

\[
\frac{1}{4} \cdot \frac{3}{4} = \frac{3}{12} \quad \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{12} \quad \frac{1}{6} \cdot \frac{2}{6} = \frac{2}{12}
\]

Add or subtract as usual (reduce if necessary):
\[
\frac{1}{4} + \frac{2}{6} - \frac{1}{12} + \frac{8}{12} = \frac{9}{12} = \frac{3}{4}
\]
**Multiplication**

When multiplying fractions, you do not need a common denominator. Just multiply the numerators and denominators.

Examples:

\[
\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6} \quad \frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20}
\]

Sometimes you will need to reduce. You may reduce before or after multiplying, though it is often easier to reduce first. Cancel and one factor in the numerator of either fraction will the same factor in the denominator of either fraction.

Examples:

\[
\frac{1}{3} \times \frac{1}{5} = \frac{1 \times 3}{3 \times 5} = \frac{1}{5} \quad \frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{3}{10}
\]

\[
\frac{6}{7} \times \frac{7}{9} = \frac{6 \times 7}{7 \times 9} = \frac{1}{1} = \frac{2}{1} = \frac{2}{3}
\]

**Division**

To divide fractions, flip over the second fraction and change to multiplication. The upside-down fraction is called the reciprocal.

Examples:

\[
\frac{3}{7} \div \frac{1}{2} = \frac{3}{7} \times \frac{2}{1} = \frac{3 \times 2}{7 \times 1} = \frac{6}{7}
\]

\[
\frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20}
\]